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Epistemic GDL: A Logic for Representing and Reasoning about Imperfect Information Games

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Outline

Background: General Game Playing (GGP)

AI programs are able to play more than one games successfully.

General Game Player

Systems

- able to understand the rules of previously unknown games.
- able to learn to play these games well without human intervention.

Official Languages

- General Game Description Language (GDL)
  - machine-processable logical language for representing the rules of arbitrary finite games [Love et al., 2006].
- GDL-II for imperfect information games
  - describe any extensive-form game with randomness and imperfect information [Thielscher, 2011].

Motivation

Challenge

- Playing games with imperfect information poses an intricate reasoning challenge for players.
- GDL-II is purely a game descriptive language but does not provide a reasoning facility.

Related Work

mostly embedding GDL-II into a logical system, such as

- Situation Calculus
  [Schiffel and Thielscher, 2011, Schiffel and Thielscher, 2014]
- Alternating-time Temporal Epistemic Logic (ATEL)
  [Ruan and Thielscher, 2012]

use the inference mechanics of the targeting logics

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problem

- High expressivity incurs high complexity
- Not tailor-made for GDL or GDL-II

Epistemic GDL (EGDL)

The language of EGDL consists of

- N: a non-empty finite set of agents.
- A^r: a non-empty finite set of actions for each agent $r \in N$. $\mathcal{A} = \bigcup_{r \in N} A^r$.
- Φ: a non-empty finite set of propositional variables.
- $\neg$ and $\wedge$
- $initial$, $terminal$, $wins(r)$, $legal(a^r)$ and $does(a^r)$ for $r \in N$, $a^r \in A^r$.
- $\bigcirc\varphi$
- the standard epistemic operators [Fagin et al., 2003]:
  - $\mathsf{K}_r\varphi$ means "*agent r knows* φ".
  - $\mathsf{C}\varphi$ means "φ *is common knowledge among all the agents*".

Syntax

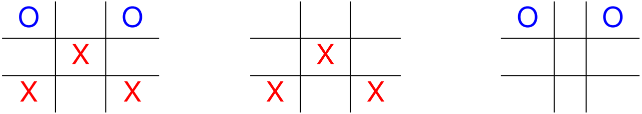
$$\varphi ::= p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \mathit{initial} \mid \mathit{terminal} \mid \mathit{wins}(r) \mid \mathit{legal}(a^r) \mid$$

$$\mathit{does}(a^r) \mid \bigcirc\varphi \mid \mathsf{K}_r\varphi \mid \mathsf{C}\varphi$$

where $p \in \Phi$, $r \in N$ and $a^r \in A^r$.

Abbreviation: $\mathsf{E}\varphi =_{def} \bigwedge_{r \in N} \mathsf{K}_r\varphi$.

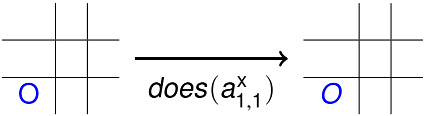
Example: Krieg-Tictactoe [Schiffel and Thielscher, 2011]



Each player can

- see her own marks, but not her opponent's.
- know turn-taking and her own available actions.

Rules of Krieg-Tictactoe



$p^o_{1,1}$ $turn(x)$ $p^o_{1,1}$ $tried(a^x_{1,1})$ $turn(o)$

Parameters

- $N_{KT} = \{x, o\}$;
- $A^r_{KT} = \{a^r_{i,j} \ : \ 1 \leqslant i, j \leqslant 3\} \cup \{noop^r\}$;
- $\Phi_{KT} = \{p^r_{i,j}, tried(a^r_{i,j}), turn(r) \ : \ r \in \{x, o\}$ and $1 \leqslant i, j \leqslant 3\}$.

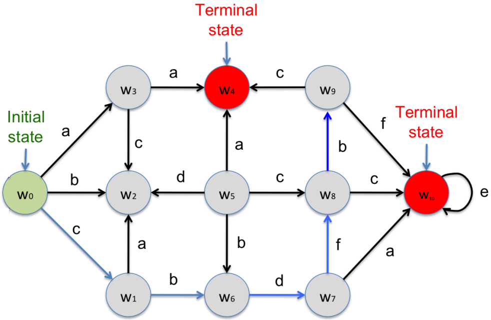
Description of Krieg-Tictactoe

1. $initial \leftrightarrow turn(\mathrm{x}) \wedge \neg turn(\mathrm{o}) \wedge \bigwedge_{i,j=1}^{3}(\neg(p_{i,j}^{\mathrm{x}} \vee p_{i,j}^{\mathrm{o}}) \wedge \neg(tried(a_{i,j}^{\mathrm{x}}) \vee tried(a_{i,j}^{\mathrm{o}})))$
2. $wins(r) \leftrightarrow$ $(\bigvee_{i=1}^{3} \bigwedge_{l=0}^{2} p_{i,l+l}^{r}) \vee (\bigvee_{j=1}^{3} \bigwedge_{l=0}^{2} p_{l+l,j}^{r}) \vee (\bigwedge_{l=0}^{2} p_{1+l,1+l}^{r}) \vee (\bigwedge_{l=0}^{2} p_{1+l,3-l}^{r})$
3. $teminal \leftrightarrow wins(\mathrm{x}) \vee wins(\mathrm{o}) \vee \bigwedge_{i,j=1}^{3}(p_{i,j}^{\mathrm{x}} \vee p_{i,j}^{\mathrm{o}})$
4. $turn(r) \wedge \neg terminal \rightarrow \bigcirc \neg turn(r) \wedge \bigcirc turn(-r)$
5. $legal(noop^{r}) \leftrightarrow turn(-r) \vee terminal$
6. $legal(a_{i,j}^{r}) \leftrightarrow turn(r) \wedge \neg p_{i,j}^{r} \wedge \neg tried(a_{i,j}^{r}) \wedge \neg terminal$
7. $\bigcirc p_{i,j}^{r} \leftrightarrow p_{i,j}^{r} \vee (does(a_{i,j}^{r}) \wedge \neg(p_{i,j}^{\mathrm{x}} \vee p_{i,j}^{\mathrm{o}}))$
8. $\bigcirc tried(a_{i,j}^{r}) \leftrightarrow tried(a_{i,j}^{r}) \vee (does(a_{i,j}^{r}) \wedge p_{i,j}^{-r})$
9. $does(a_{i,j}^{r}) \rightarrow \mathrm{K}_r(does(a_{i,j}^{r}))$
10. $initial \rightarrow \mathrm{E}initial$
11. $(turn(r) \rightarrow \mathrm{E}turn(r)) \wedge (\neg turn(r) \rightarrow \mathrm{E}\neg turn(r))$
12. $(p_{i,j}^{r} \rightarrow \mathrm{K}_r p_{i,j}^{r}) \wedge (\neg p_{i,j}^{r} \rightarrow \mathrm{K}_r \neg p_{i,j}^{r})$
13. $(tried(a_{i,j}^{r}) \rightarrow \mathrm{K}_r tried(a_{i,j}^{r})) \wedge (\neg tried(a_{i,j}^{r}) \rightarrow \mathrm{K}_r \neg tried(a_{i,j}^{r}))$

Epistemic State Transition Model

State Transition Model + Epistemic Relations

State Transition Model



Epistemic State Transition Model

An epistemic state transition (ET) model M is a tuple $(W, I, T, \{R_r\}_{r\in N}, g, \{L_r\}_{r\in N}, U, \pi)$, where

- W is a nonempty set of *states*.
- $I \subseteq W$ is the set of *initial* states.
- $T \subseteq W \backslash I$ is the set of *terminal* states.
- $R_r \subseteq W \times W$ is an equivalence relation for agent r.
- $g : N \to 2^W$ is a *goal* function.
- $L_r \subseteq W \times A^r$ is a *legality* relation.
- $U : W \times \prod_{r\in N} A^r \hookrightarrow W \backslash I$ is an *update* function.
- $\pi : W \to 2^\Phi$ is a *valuation function*.

Basic Assumptions for ET-Models

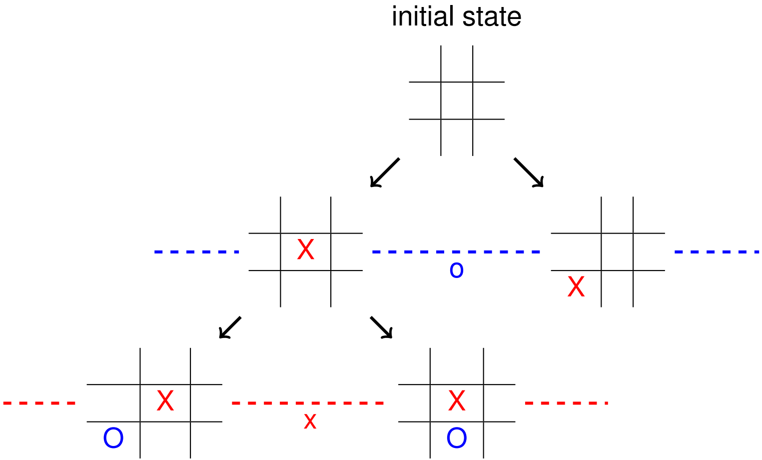
Let $L_r(w)$ denote the set of all legal actions for agent r at w. Then

(i) $L_r(w) \neq \emptyset$ for any $r \in N$ and $w \in W \backslash T$;

(ii) $L_r(w) = \{noop^r\}$ for any $r \in N$ and $w \in T$.

(iii) $U(w, \langle noop^r \rangle_{r \in N}) = w$ for any $w \in T$.

ET-Model of Krieg-Tictactoe



Complete Path

A complete path δ is an infinite sequence of states and joint actions $w_0 \xrightarrow{d_1} w_1 \xrightarrow{d_2} w_2 \cdots \xrightarrow{d_j} \cdots$ such that for all $j \geqslant 1$ and any $r \in N$,

1. $w_0 \in I$, $w_j \notin I$;
2. $d_j(r) \in L_r(w_{j-1})$;
3. $w_j = U(w_{j-1}, d_j)$, and
4. if $w_j \in T$, then $w_j = w_{j+1}$.

Imperfect Recall

Consider two complete paths

$$\delta := w_0 \xrightarrow{d_1} \cdots \xrightarrow{d_j} w_j \xrightarrow{d_{j+1}} \cdots$$

$$\delta' := w'_0 \xrightarrow{d'_1} \cdots \xrightarrow{d'_j} w'_j \xrightarrow{d'_{j+1}} \cdots .$$

δ and δ' are imperfect recall equivalent for player r at stage j, written $\delta \approx^j_r \delta'$, iff $w_j R_r w'_j$.

Semantics

A formula φ is true at a stage j of a complete path δ under M, denoted by $M, \delta, j \models \varphi$, if

$M, \delta, j \models p$	iff	$p \in \pi(\delta[j])$
$M, \delta, j \models \neg\varphi$	iff	$M, \delta, j \not\models \varphi$
$M, \delta, j \models \varphi_1 \wedge \varphi_2$	iff	$M, \delta, j \models \varphi_1$ and $M, \delta, j \models \varphi_2$
$M, \delta, j \models initial$	iff	$\delta[j] \in I$
$M, \delta, j \models terminal$	iff	$\delta[j] \in T$
$M, \delta, j \models wins(r)$	iff	$\delta[j] \in g(r)$
$M, \delta, j \models legal(a^r)$	iff	$(\delta[j], a^r) \in L_r$
$M, \delta, j \models does(a^r)$	iff	$\theta_r(\delta, j) = a^r$
$M, \delta, j \models \bigcirc\varphi$	iff	$M, \delta, j+1 \models \varphi$
$M, \delta, j \models \mathsf{K}_r\varphi$	iff	for any $\delta' \approx_r^j \delta$, $M, \delta', j \models \varphi$
$M, \delta, j \models \mathsf{C}\varphi$	iff	for any $\delta' \approx_N^j \delta'$, $M, \delta', j \models \varphi$

where $\approx_N^j$ is its transitive closure of $\bigcup_{r \in N} \approx_r^j$.

Epistemic Properties

(1) $initial \rightarrow Cinitial$

(2) $legal(a^r) \rightarrow K_r(legal(a^r))$

(3) $does(a^r) \rightarrow K_r(does(a^r))$

(4) $wins(r) \rightarrow K_r(wins(r))$

(5) $terminal \rightarrow Cterminal$

Note

- Formula (2): a semantic property yet with no syntactic expression in ATEL [Ågotnes, 2006];
- Formula (3): the "uniform" property of actions with no syntactic expression in ATEL [van der Hoek and Wooldridge, 2003].

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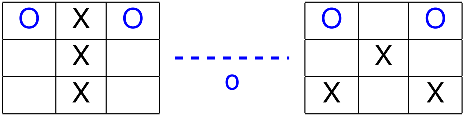
Krieg-Tictactoe satisfies all the properties, except (5).



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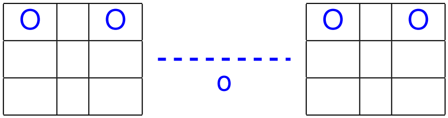


Epistemic Properties

(1) $initial \rightarrow \mathsf{C}initial$

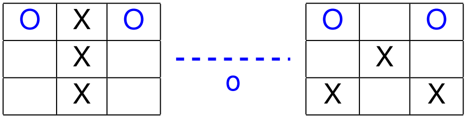
(2) $legal(a^r) \rightarrow \mathsf{K}_r(legal(a^r))$

(3) $does(a^r) \rightarrow \mathsf{K}_r(does(a^r))$

(4) $wins(r) \rightarrow \mathsf{K}_r(wins(r))$

(5) $terminal \rightarrow \mathsf{C}terminal$

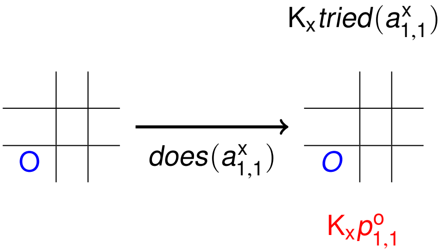
Krieg-Tictactoe satisfies all the properties, except (5).



a terminal state

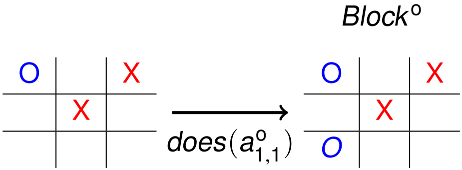
a non-terminal state

Reasoning about Game Rules



- $does(a^r_{i,j}) \to \bigcirc K_r(p^r_{i,j} \vee tried(a^r_{i,j}))$
- $K_r tried(a^r_{i,j}) \to K_r p^{-r}_{i,j}$

Strategic Reasoning



- $check^r = K_r(does(a^r_{i,j}) \wedge \bigcirc wins(r)) \rightarrow does(a^r_{i,j})$
- $block^r = K_r \bigcirc (does(a^{-r}_{i,j}) \wedge \bigcirc wins(-r)) \rightarrow does(a^r_{i,j})$

Model Checking

The model checking problem for EGDL:
Given an EGDL-formula φ, an ET-model M, a complete path δ of M and a stage j on δ, determining whether $M, \delta, j \models \varphi$ or not.

Model Checking

Complexity

The model-checking problem of EGDL is Θ_2^p-hard yet in Δ_2^p.

- Θ_2^p: reduce the validity problem of Carnap's modal logic [Gottlob, 1995].
- Δ_2^p: develop a model-checking algorithm.

Both lie in the second level of the polynomial hierarchy.

Conclusions

- Proposed an epistemic extension of GDL for imperfect information games with imperfect recall players.
- Demonstrated its expressiveness and investigated its model-checking problem.

Make a good balance between expressive power and computational efficiency.

- Future Work
  - Other Memory Types: State-based perfect recall, Action-based perfect recall, Perfect recall
  - Game Equivalence
  - Strategy Representation and Revision

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