Epistemic GDL: A Logic for Representing and Reasoning about Imperfect Information Games

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Outline

1. Introduction
2. Syntax and Semantics
3. Epistemic and Strategic Reasoning
4. Model Checking
5. Conclusions
Background: General Game Playing (GGP)

AI programs are able to play more than one game successfully.
General Game Player

- able to understand the rules of previously unknown games.
- able to learn to play these games well without human intervention.
Official Languages

- **General Game Description Language (GDL)**
  - machine-processable logical language for representing the rules of arbitrary finite games [Love et al., 2006].

- **GDL-II for imperfect information games**
  - describe any extensive-form game with randomness and imperfect information [Thielscher, 2011].
Motivation

Challenge

- Playing games with imperfect information poses an intricate reasoning challenge for players.
- GDL-II is purely a game descriptive language but does not provide a reasoning facility.
Related Work

mostly embedding GDL-II into a logical system, such as

- Situation Calculus
  [Schiffel and Thielscher, 2011, Schiffel and Thielscher, 2014]

- Alternating-time Temporal Epistemic Logic (ATEL)
  [Ruan and Thielscher, 2012]

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**Problem**

- High expressivity incurs high complexity
- Not tailor-made for GDL or GDL-II
Epistemic GDL (EGDL)

The language of EGDL consists of

- \( N \): a non-empty finite set of agents.
- \( A^r \): a non-empty finite set of actions for each agent \( r \in N \). \( A = \bigcup_{r \in N} A^r \).
- \( \Phi \): a non-empty finite set of propositional variables.
- \( \neg \) and \( \land \)
- \textit{initial}, \textit{terminal}, \textit{wins}(r), \textit{legal}(a^r) \text{ and } \textit{does}(a^r) \text{ for } r \in N, a^r \in A^r.
- \( \Box \varphi \)

the standard epistemic operators [Fagin et al., 2003]:

- \( K_r \varphi \) means “agent \( r \) knows \( \varphi \).”
- \( C \varphi \) means “\( \varphi \) is common knowledge among all the agents”.
Syntax

\( \varphi ::= p \mid \neg \varphi \mid \varphi \land \varphi \mid \text{initial} \mid \text{terminal} \mid \text{wins}(r) \mid \text{legal}(a^r) \mid \)

\( \text{does}(a^r) \mid \bigcirc \varphi \mid K_r \varphi \mid C \varphi \)

where \( p \in \Phi, \ r \in N \) and \( a^r \in A^r \).

Abbreviation: \( E \varphi :=_{\text{def}} \bigwedge_{r \in N} K_r \varphi \).
Example: Krieg-Tictactoe [Schiffel and Thielscher, 2011]

Each player can
- see her own marks, but not her opponent’s.
- know turn-taking and her own available actions.
Rules of Krieg-Tictactoe

\[
\begin{array}{c|c|c}
| & | & | \\
O & | & O \\
| & does(a^x_{1,1}) & | \\
\end{array}
\]

\[p^o_{1,1} \text{ turn}(x) \quad p^o_{1,1} \text{ tried}(a^x_{1,1}) \text{ turn}(o)\]

**Parameters**

- \(N_{KT} = \{x, o\}\);
- \(A^r_{KT} = \{a^r_{i,j} : 1 \leq i, j \leq 3\} \cup \{noop^r\}\);
- \(\Phi_{KT} = \{p^r_{i,j}, tried(a^r_{i,j}), turn(r) : r \in \{x, o\} \text{ and } 1 \leq i, j \leq 3\}\).
Description of Krieg-Tictactoe

1. $\text{initial} \leftrightarrow \text{turn}(x) \land \neg \text{turn}(o) \land \bigwedge_{i,j=1}^{3} (-p_{i,j}^x \lor p_{i,j}^o) \land \neg (\text{tried}(a_{i,j}^x) \lor \text{tried}(a_{i,j}^o))$

2. $\text{wins}(r) \leftrightarrow$
   $$(\bigvee_{i=1}^{3} \bigwedge_{l=0}^{2} p_{i,l+l}^r) \lor (\bigvee_{j=1}^{3} \bigwedge_{l=0}^{2} p_{l+j,i}^r) \lor (\bigwedge_{l=0}^{2} p_{1+l,1+l}^r) \lor (\bigwedge_{l=0}^{2} p_{1+l,3-l}^r)$$

3. $\text{terminal} \leftrightarrow \text{wins}(x) \lor \text{wins}(o) \lor \bigwedge_{i,j=1}^{3} (p_{i,j}^x \lor p_{i,j}^o)$

4. $\text{turn}(r) \land \neg \text{terminal} \rightarrow \Box \neg \text{turn}(r) \land \Box \text{turn}(\neg r)$

5. $\text{legal}(\text{noop}^r) \leftrightarrow \text{turn}(\neg r) \lor \text{terminal}$

6. $\text{legal}(a_{i,j}^r) \leftrightarrow \text{turn}(r) \land \neg p_{i,j}^r \land \neg \text{tried}(a_{i,j}^r) \land \neg \text{terminal}$

7. $\Box p_{i,j}^r \leftrightarrow p_{i,j}^r \lor (\text{does}(a_{i,j}^r) \land \neg (p_{i,j}^x \lor p_{i,j}^o))$

8. $\Box \text{tried}(a_{i,j}^r) \leftrightarrow \text{tried}(a_{i,j}^r) \lor (\text{does}(a_{i,j}^r) \land p_{i,j}^r)$

9. $\text{does}(a_{i,j}^r) \rightarrow K_r(\text{does}(a_{i,j}^r))$

10. $\text{initial} \rightarrow E\text{initial}$

11. $$(\text{turn}(r) \rightarrow E\text{turn}(r)) \land (\neg \text{turn}(r) \rightarrow E\neg \text{turn}(r))$$

12. $$(p_{i,j}^r \rightarrow K_r p_{i,j}^r) \land (\neg p_{i,j}^r \rightarrow K_r \neg p_{i,j}^r)$$

13. $$(\text{tried}(a_{i,j}^r) \rightarrow K_r \text{tried}(a_{i,j}^r)) \land (\neg \text{tried}(a_{i,j}^r) \rightarrow K_r \neg \text{tried}(a_{i,j}^r))$$
Epistemic State Transition Model

State Transition Model + Epistemic Relations
State Transition Model
An epistemic state transition (ET) model $M$ is a tuple $(W, I, T, \{R_r\}_{r \in N}, g, \{L_r\}_{r \in N}, U, \pi)$, where

- $W$ is a nonempty set of states.
- $I \subseteq W$ is the set of initial states.
- $T \subseteq W \setminus I$ is the set of terminal states.
- $R_r \subseteq W \times W$ is an equivalence relation for agent $r$.
- $g : N \to 2^W$ is a goal function.
- $L_r \subseteq W \times A^r$ is a legality relation.
- $U : W \times \prod_{r \in N} A^r \hookrightarrow W \setminus I$ is an update function.
- $\pi : W \to 2^\Phi$ is a valuation function.
Let $L_r(w)$ denote the set of all legal actions for agent $r$ at $w$. Then

(i) $L_r(w) \neq \emptyset$ for any $r \in N$ and $w \in W \setminus T$;

(ii) $L_r(w) = \{noop^r\}$ for any $r \in N$ and $w \in T$.

(iii) $U(w, \langle noop^r \rangle_{r \in N}) = w$ for any $w \in T$. 
ET-Model of Krieg-Tictactoe
A complete path $\delta$ is an infinite sequence of states and joint actions $w_0 \xrightarrow{d_1} w_1 \xrightarrow{d_2} w_2 \cdots \xrightarrow{d_j} \cdots$ such that for all $j \geq 1$ and any $r \in N$,

1. $w_0 \in I, w_j \notin I$;
2. $d_j(r) \in L_r(w_{j-1})$;
3. $w_j = U(w_{j-1}, d_j)$, and
4. if $w_j \in T$, then $w_j = w_{j+1}$.
Consider two complete paths

\[
\delta := w_0 \xrightarrow{d_1} \cdots \xrightarrow{d_j} w_j \xrightarrow{d_{j+1}} \cdots \\
\delta' := w'_0 \xrightarrow{d'_1} \cdots \xrightarrow{d'_j} w'_j \xrightarrow{d'_{j+1}} \cdots .
\]

\(\delta\) and \(\delta'\) are imperfect recall equivalent for player \(r\) at stage \(j\), written \(\delta \simeq^j_r \delta'\), iff \(w_j R_r w'_j\).
A formula $\varphi$ is true at a stage $j$ of a complete path $\delta$ under $M$, denoted by $M, \delta, j \models \varphi$, if

- $M, \delta, j \models p$ iff $p \in \pi(\delta[j])$
- $M, \delta, j \models \neg \varphi$ iff $M, \delta, j \not\models \varphi$
- $M, \delta, j \models \varphi_1 \land \varphi_2$ iff $M, \delta, j \models \varphi_1$ and $M, \delta, j \models \varphi_2$
- $M, \delta, j \models initial$ iff $\delta[j] \in I$
- $M, \delta, j \models terminal$ iff $\delta[j] \in T$
- $M, \delta, j \models wins(r)$ iff $\delta[j] \in g(r)$
- $M, \delta, j \models legal(a^r)$ iff $(\delta[j], a^r) \in L_r$
- $M, \delta, j \models does(a^r)$ iff $\theta_r(\delta, j) = a^r$
- $M, \delta, j \models \Box \varphi$ iff $M, \delta, j + 1 \models \varphi$
- $M, \delta, j \models K_r \varphi$ iff for any $\delta' \approx^j_r \delta$, $M, \delta', j \models \varphi$
- $M, \delta, j \models C \varphi$ iff for any $\delta' \approx^j_N \delta'$, $M, \delta', j \models \varphi$

where $\approx^j_N$ is its transitive closure of $\bigcup_{r \in N} \approx^j_r$. 
Epistemic Properties

(1) \textit{initial} \rightarrow C_{\text{initial}} \quad (2) \text{legal}(a') \rightarrow K_r(\text{legal}(a'))

(3) \text{does}(a') \rightarrow K_r(\text{does}(a')) \quad (4) \text{wins}(r) \rightarrow K_r(\text{wins}(r))

(5) \textit{terminal} \rightarrow C_{\text{terminal}}

Note

- Formula (2): a semantic property yet with no syntactic expression in ATEL [Ågotnes, 2006];
- Formula (3): the “uniform” property of actions with no syntactic expression in ATEL [van der Hoek and Wooldridge, 2003].
Epistemic Properties

(1) $\text{initial} \rightarrow C_{\text{initial}}$
(2) $\text{legal}(a') \rightarrow K_r(\text{legal}(a'))$
(3) $\text{does}(a') \rightarrow K_r(\text{does}(a'))$
(4) $\text{wins}(r) \rightarrow K_r(\text{wins}(r))$
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Krieg-Tictactoe satisfies all the properties, except (5).
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Epistemic Properties

(1) $initial \rightarrow C_{initial}$

(2) $legal(a^r) \rightarrow K_r(legal(a^r))$

(3) $does(a^r) \rightarrow K_r(does(a^r))$

(4) $wins(r) \rightarrow K_r(wins(r))$

(5) $terminal \rightarrow C_{terminal}$

Krieg-Tictactoe satisfies all the properties, except (5).

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a terminal state  a non-terminal state
Reasoning about Game Rules

\[ K_x tried(a^x_{1,1}) \]

\[ \text{O} \quad \text{does}(a^x_{1,1}) \quad \text{O} \]

\[ K_x p^0_{1,1} \]

- \( does(a^r_{i,j}) \rightarrow \bigcirc K_r (p^r_{i,j} \lor tried(a^r_{i,j})) \)

- \( K_r tried(a^r_{i,j}) \rightarrow K_r p^r_{i,j} \)
Strategic Reasoning

\[ \text{Block}^0 \]

\[
\begin{array}{c|c|c}
\text{O} & \text{X} & \text{X} \\
\hline
\text{X} & \text{X} & \text{O} \\
\hline
\text{does}(a_{1,1}^0) & \text{O} & \text{X} \\
\end{array}
\]

- \( \text{check}^r = K_r(\text{does}(a_{i,j}^r) \land \bigcirc \text{wins}(r)) \rightarrow \text{does}(a_{i,j}^r) \)
- \( \text{block}^r = K_r(\bigcirc(\text{does}(a_{i,j}^{-r}) \land \bigcirc \text{wins}(\neg r)) \rightarrow \text{does}(a_{i,j}^r) \)
The model checking problem for EGDL:
Given an EGDL-formula $\varphi$, an ET-model $M$, a complete path $\delta$ of $M$ and a stage $j$ on $\delta$, determining whether $M, \delta, j \models \varphi$ or not.
The model-checking problem of EGDL is $\Theta_2^p$-hard yet in $\Delta_2^p$.

- $\Theta_2^p$: reduce the validity problem of Carnap’s modal logic [Gottlob, 1995].
- $\Delta_2^p$: develop a model-checking algorithm.

Both lie in the second level of the polynomial hierarchy.
Conclusions

- Proposed an epistemic extension of GDL for imperfect information games with imperfect recall players.
- Demonstrated its expressiveness and investigated its model-checking problem.

Make a good balance between expressive power and computational efficiency.

Future Work
- Other Memory Types: State-based perfect recall, Action-based perfect recall, Perfect recall
- Game Equivalence
- Strategy Representation and Revision


